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Vacuum consolidation effects on lateral yield of soft clays as applied to road and railway embankment

Buddhima Indraratna
University of Wollongong, indra@uow.edu.au

Cholachat Rujikiatkamjorn
University of Wollongong, cholacha@uow.edu.au

G. McIntosh
Douglas Partners Pty Ltd

A. Balasubramaniam
Griffith University

VACUUM CONSOLIDATION EFFECTS ON LATERAL YIELD OF SOFT CLAYS AS APPLIED TO ROAD AND RAILWAY EMBANKMENT

Buddhima Indraratna
Professor of Civil Engineering, School of Civil, Mining and Environmental Engineering, University of Wollongong, Wollongong City, NSW 2522, Australia

Cholachat Rujkiatkamjorn
Research Associate, School of Civil, Mining and Environmental Engineering, University of Wollongong, NSW 2522, Australia

Geoffrey McIntosh
Principal Geotechnical Engineer, Douglas Partners Pty Ltd, Unanderra, NSW Australia

A.S. Balasubramaniam
Professor, School of Engineering, Griffith University, Gold Coast, QLD, 4111, Australia
(Formerly, at Asian Institute of Technology, Bangkok, Thailand)

ABSTRACT

The use of vertical drains with vacuum preloading is considered as the most effective and economical method for improving soft clays (normally consolidated to lightly over-consolidated) to eliminate settlements of the permanent infrastructure. Application of vacuum pressure via prefabricated vertical drains promotes radial flow consolidation enhancing the shear strength of the compressed ground. In this paper, the mechanisms of the vacuum preloading system based on current practices are described through the selection of the important design parameters. The equivalent plane strain solution for both Darcian and non-Dracian flow are presented to predict the excess pore pressures, lateral and vertical displacements. The numerical analyses incorporating equivalent plane strain solutions were performed to predict the soil responses based on two selected case histories in Thailand, one with vacuum and the other without vacuum application. The research findings provided insight as to which of the above aspects needed to be simulated accurately in numerical modelling. It is found that the accuracy of modelling depends on the correct selection of the constitutive model applied in the numerical analysis. Finally, a parametric study of the combination of vacuum and surcharge preloading was conducted to demonstrate how any excessive lateral displacements can be avoided.

Keywords: Case histories, Numerical analysis, Vacuum consolidation Vertical drains,

INTRODUCTION

Many coastal regions of Australia and Southeast Asia contain very soft clays, which have poor geotechnical properties such as, low bearing capacity and high compressibility. Buildings, port and transport infrastructure, highway and rail embankments have been affected by the settlement and lateral movement of soft clays, in the absence of appropriate ground improvement prior to construction (Indraratna and Chu, 2005). The installation of vertical drains and then pre-loading the ground surface by placing a few meters of temporary fill surcharge (i.e. prior to construction of the main structure) facilitates dissipation of internal water pressure in the soil, thereby accelerating consolidation (Indraratna et al. 1994). Once the clay foundation has experienced initial settlement (primary consolidation), the subsequent soil deformation after constructing the permanent structure will be much less, thereby providing it with increased stability.

PURPOSE AND APPLICATION OF VERTICAL DRAINS

Various types of vertical drains including sand drains, sand compaction piles, prefabricated vertical drains (geosynthetic) and gravel piles have been commonly used in the past. It was conventional to use sand or gravel filled boreholes as vertical drains, such as sand/gravel piles, stone columns etc. (Jamiolkowski, 1983). Due to the advancement in polymer technology, synthetic drains have replaced the need for using natural aggregates. Moreover, band-shaped prefabricated vertical drains (PVD) can be installed more quickly with a steel mandrel (Fig. 1). A steel anchor is located at the mandrel tip to prevent soil entering and act as an anchor to keep it at the bottom of the cell (Fig. 2). The mandrel is then removed leaving the drain in place. PVDs usually consist of a porous plastic core lined by a filter (Fig. 3). The most common band shaped drains have cross section dimensions of 100mm × 4mm. When installed in the ground, they shorten the drainage path length (radial flow), thereby preventing the build up of excess pore water pressure, thus reducing the risk of failure (Hansbo, 1981). Since undrained soft clay can undergo high lateral displacement at or close to failure, an important stabilising role of PVD is that they can reduce the lateral soil movement by promoting initial compression of the soil (Indraratna & Redana, 2000). When the lateral displacements decrease, the stresses imposed on adjacent structures such as pipelines also decrease, minimising potential damage.

A typical instrumented vertical drain scheme is presented in Fig. 4. The site is initially prepared by removing vegetation and surface debris, and grading the ground. It is useful to minimize the disturbance to the weathered surface crust which may provide at least some resistance against the lateral movement just beneath the embankment base, in the same manner as geogrid. The sand blanket provides a sound working platform and provides the drainage for water from PVDs. The drainage function of the sand blanket may be equipped with horizontal drains on the surface.

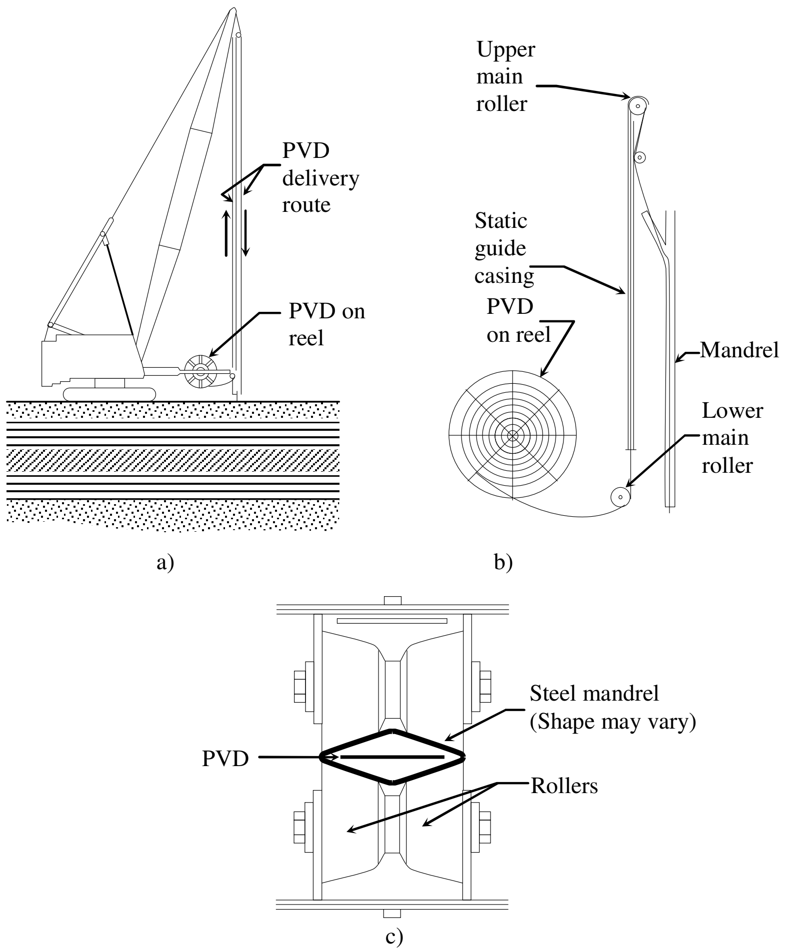


Fig. 1 PVD installation a) crane mounted installation rig, b) drain delivery arrangement, c) cross section of mandrel and drain (after Koerner, 1987)

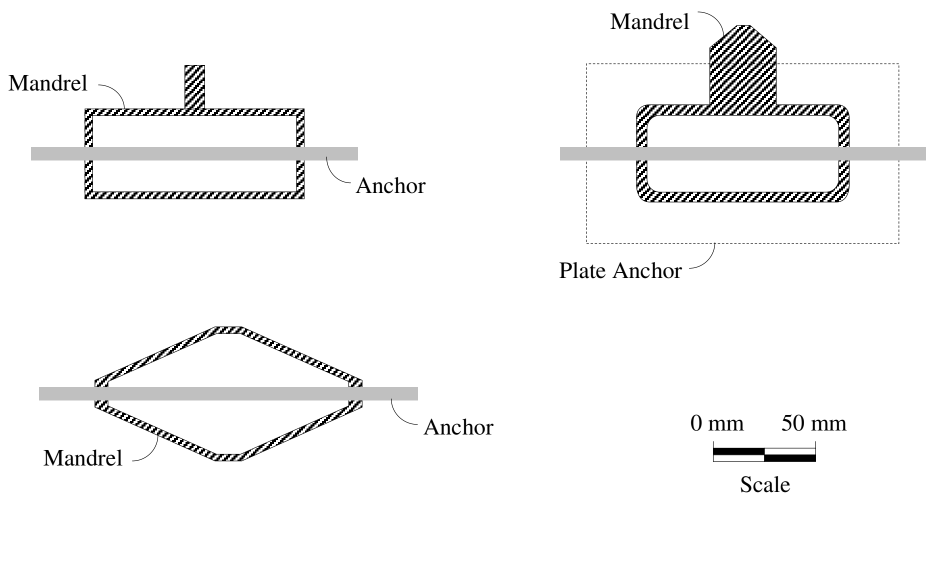


Fig. 2 Examples of mandrel shapes (Saye, 2001)

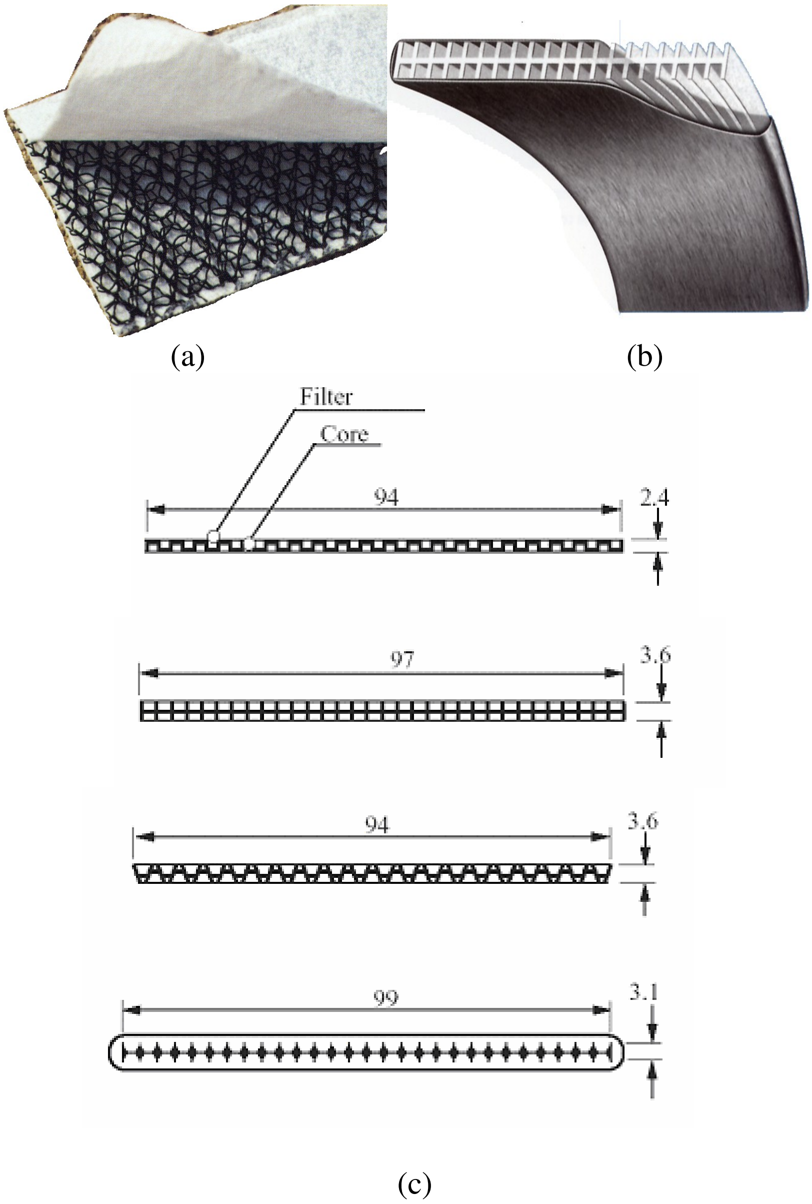


Fig. 3 Typical types of PVD (a) Colbond drain (b) Mebra drain (company brochure) (c) Cross-section of various versions of PVDs (After Chai et al. 2001)

Field instrumentation for monitoring and evaluating the performance of embankments is necessary to ensure the performance of embankment. The data will assist in improving the settlement predictions and for providing guidelines for future projects. Field instrumentation can be categorised into two groups (Bo et al., 2003). The first group is used to prevent sudden failures during construction (e.g. settlement plates, inclinometers and piezometers), whereas the second group is used to record changes in the rate of settlement and excess pore pressure during loading stages (e.g. multilevel settlement gauges and piezometers).

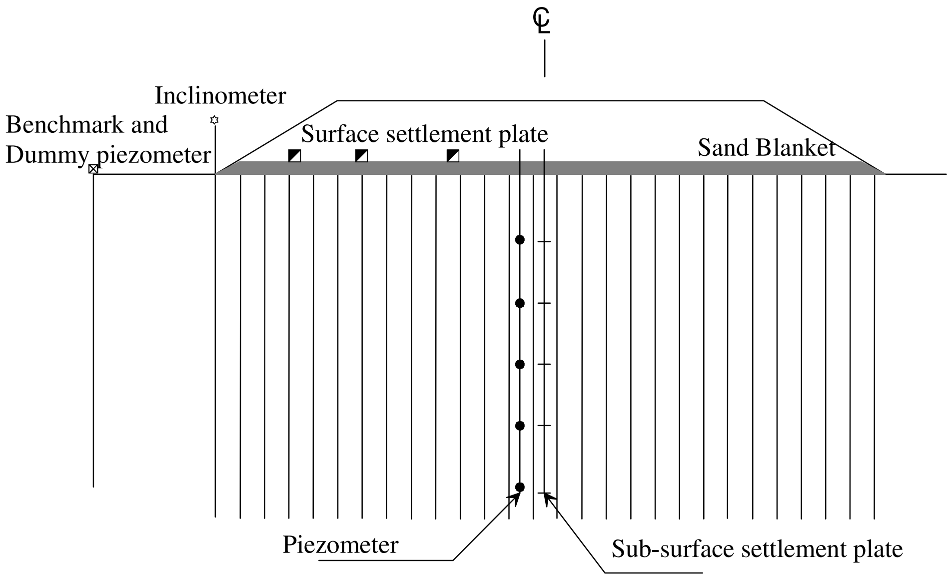


Fig. 4 Typical vertical cross section of vertical drain system (Indraratna et al. 2005b)

PRINCIPLES OF VERTICAL DRAIN INCORPORATING VACUUM PRELOADING

When the use of surcharge loading alone is too slow or inappropriate for the site and the required load would result in an embankment of unsafe height, or there is no access to suitable fill material, it is necessary to use more refined techniques instead of, or in combination with surcharge loading. One of the popular methods is applying a vacuum to the top of soil surface. An external negative load is applied to the soil surface in the form of vacuum through a sealed membrane system (Kjellman, 1952). A higher effective stress is achieved by the negative pore water pressure, while the total stress remains constant (Fig. 5). When vacuum preloading is propagated downwards via PVDs, the surrounding soil tends to move inward, while the conventional surcharge loading causes outward lateral flow. The result is a reduction of the outward lateral displacements, thereby reducing the risk of damage to adjacent structures and sudden undrained failure. It is important to ensure that the site to be treated is totally sealed and isolated from any surrounding permeable soils to avoid vacuum leakage that adversely affects the vacuum performance (Indraratna et al. 2005e).

Currently, there are two types of vacuum preloading systems, which are utilised in the field.

A. Membrane system (e.g. Menard Drain System)

After installing the PVDs by a steel mandrel and placing the sand blanket (i.e. top drainage layer), the installation of some horizontal drains in the transverse and longitudinal direction is desirable. Subsequently, these drains can be connected to the edge of a peripheral bentonite slurry trench, which is typically sealed by an impervious geomembrane (Fig. 6a). The membrane is placed over the sand blanket too in order to ensure an airtight region above the PVDs. The vacuum pumps are then connected to the prefabricated discharge system extending from the trenches. A key advantage of this system is that the suction head generated by the pump propagates along the soil surface and down the PVDs within the airtight domain, accelerating the dissipation of pore water pressure radially towards the PVDs as well as towards the surface. However, an obvious set back is that the efficiency of the entire system depends on the ability of the membrane to prevent any air leaks to sustain a sufficient suction head over a significant period of time (Indraratna et al. 2004).

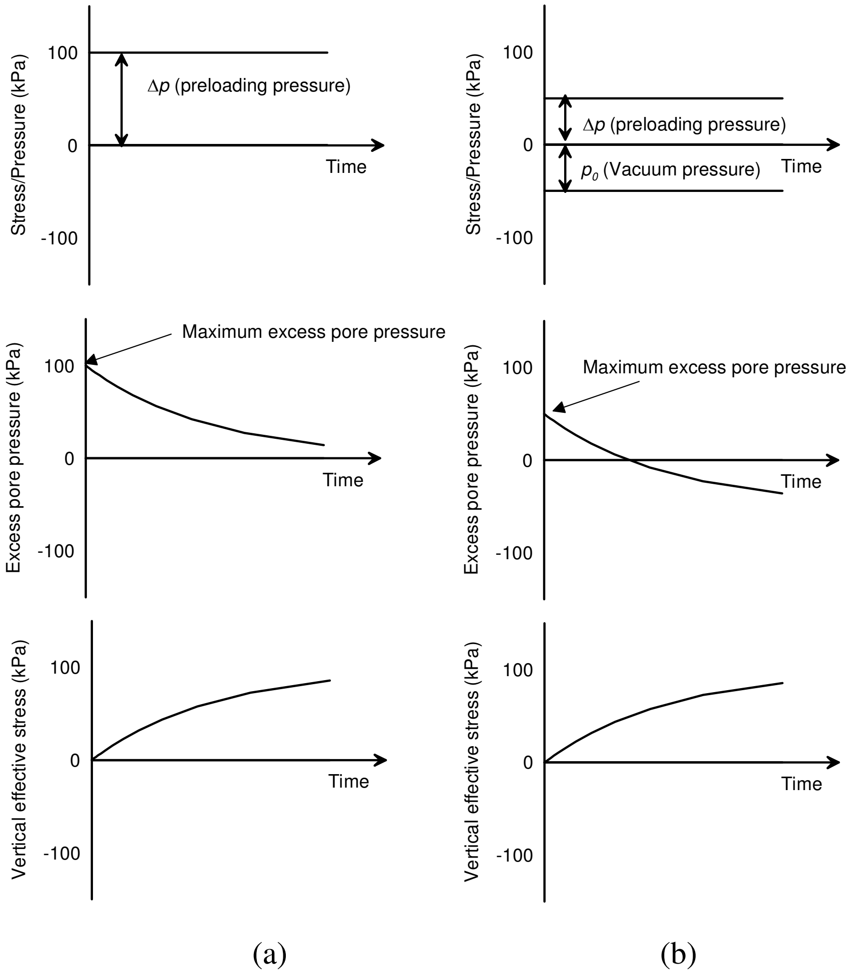
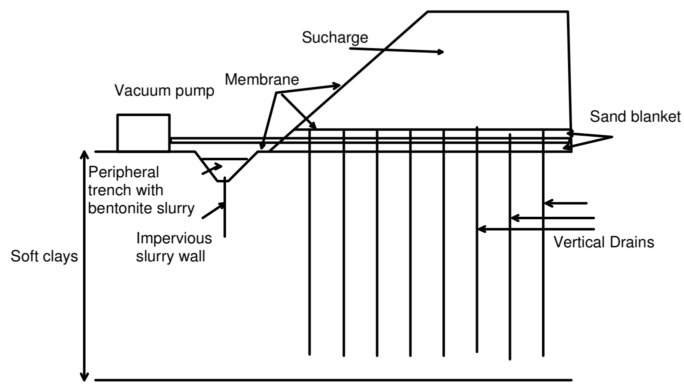


Fig. 5 Consolidation process (a) conventional loading (b) vacuum preloading assuming no vacuum loss (Indraratna et al. 2005d)

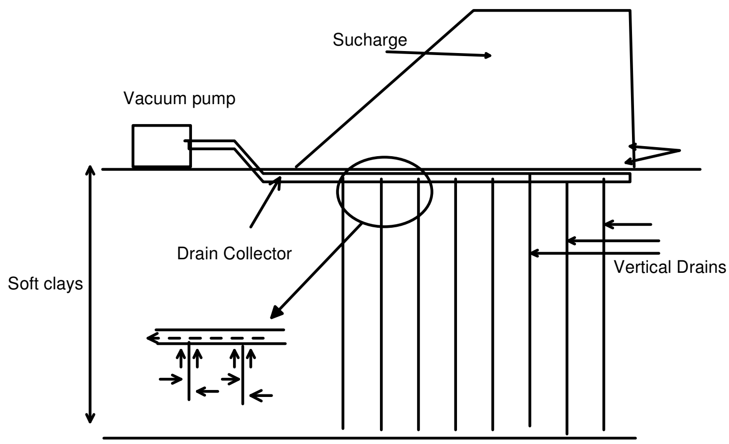
B. Membraneless system (e.g. Beau Drain System)

When an area has to be subdivided into a number of sections to facilitate the installation of the membrane, as in the Menard system, the vacuum preloading can only be carried out one section after another. This procedure is not efficient when the vacuum preloading method is used for instance in land reclamation over a large area. One way of overcoming this problem is to connect the vacuum channel directly to each individual PVD using a tubing system. In this system, each individual drain is connected directly to the drain collector (Fig. 6b). Unlike in the membrane system where any air leak can affect all the drains, in this system each drain acts independently. However, the requirement of extensive tubing for hundreds of drains can affect the installation time and cost (Seah, 2006).

Apart from the obvious inherent characteristics of these two distinctly different vacuum systems, their effectiveness also depends on the soil properties, the depth of clay layer, drain spacing, type and geometry of drains, and the vacuum pump capacity among other factors. Currently, the selection and use of these systems are based on empirical assessments with little fundamental or numerical studies, and invariably influenced strongly by tender costs and experience of the contractors.



(a)



(b)

Fig. 6 PVDs incorporating preloading system (a) Membrane system and (b) Membraneless system (Indraratna et al. 2004)

FACTORS AFFECTING VERTICAL DRAIN BEHAVIOR

For a particular soft soil condition, the effectiveness of vertical drains is dependent on (1) equivalent drain diameter; (2) filter and apparent opening size; (3) discharge capacity and well resistance; (4) smear zone; and (5) drain Unsaturation. There are uncertainties to quantifying the influence of these factors. This section concentrates on how to evaluate the design values of these parameters.

Equivalent drain diameter for band shaped drain

Analytical solutions for radial consolidation usually assume that water flows into a drain having circular cross-section. Therefore the rectangular cross section of the band drains needs to be converted to an equivalent circular one. The following equations have been proposed to determine equivalent drain diameter for a rectangular band drain with width a and thickness b:

$$d_w = 2\frac{(a+b)}{\pi} \qquad \text{(Hansbo, 1981)} \qquad (1)$$

$$d_w = \frac{a+b}{2} \qquad \text{(Atkinson and Eldred, 1981)} \qquad (2)$$

$$d_w = \left(\frac{4ab}{\pi}\right)^{0.5} \qquad \text{(Fellenius and Castonguay, 1985)} \qquad (3)$$

$$d_w = 0.5a + 0.7b \qquad \text{(Long and Corvo, 1994)} \qquad (4)$$

Equations (1) and (2) are usually used in the design. It should be noted that there is minimal difference in the consolidation rates calculated using any of the equations (Indraratna and Redana, 2000; Walker, 2006).

Filter and apparent opening size (AOS)

The porous plastic core and the filter jacket of PVD have to satisfy two basic but contrasting requirements, which are preventing the soil particles intrusion and at the same time permitting the water to pass through. A general guideline for the drain permeability is given by:

$$k_{filter} > 2\ k_{soil} \qquad (5)$$

An effective filtration can minimise soil particles from moving through the filter (Carroll, 1983). A commonly employed filtration requirement is given by:

$$\frac{O_{95}}{D_{85}} \le 2-3 \qquad (6)$$

In the above, O_{95} indicates the approximate largest particle that would effectively pass through the filter. Sieving is carried out using glass beads of successively larger diameter until 5% passes through the filter, and this size in millimeters defines the AOS, O_{95} based on ASTM D 4751. This apparent opening size (AOS) is usually taken to be less than 90 microns based on Equation (6). D_{85} indicates the diameter of clay particles corresponding to 85% passing.

The retention ability of the filter is then given by:

$$\frac{O_{50}}{D_{50}} \le 10-12 \qquad (7)$$

Filter material can also become clogged if the soil particles become trapped within the filter fabric structure. Clogging can be prevented by ensuring that (Christopher and Holtz, 1985):

$$\frac{O_{95}}{D_{15}} \geq 3 \tag{8}$$

Discharge capacity and well resistance

The discharge capacity of the prefabricated vertical drain or well resistance factor depends on its cross section, filter permeability and surrounding soil condition in the actual field (Miura and Chai, 2000). There are two main factors related to the discharge capacity of a vertical drain: the first is the evaluation of the required discharge capacity in design; the second is the measurement of drain discharge capacity (Hird et al. 1991). In this case, the discharge capacity will be a function of the volume of the core or the drain channel, the lateral earth pressure acting on the drains (Fig. 7), possible folding, bending and twisting of the drain (Fig. 8) due to large settlement, infiltration of fine soil particles through the filter, as well as the biological and chemical degradation. Incorporating the above factors, the required discharge capacity, q_{req}, is then given by (Chu et al., 2004):

$$q_{req} \geq 7.85 F_s k_h l_m^2 \tag{9}$$

where F_s, = 4-6, l_m is maximum discharged length and k_h is soil permeability. The term q_{req} can be calculated from Barron's theory of consolidation, which is given by:

$$q_{req} = \frac{\varepsilon_f U_{10} l \pi c_h}{4T_h} \tag{10}$$

The parameters ε_f is the final settlement of the soft soil equivalent to 25% of the length of the drain installed to the soft ground, U_{10} is the 10 percent degree of consolidation, l is the depth of the vertical drain, c_h is horizontal coefficient of consolidation and T_h is the time factor for horizontal (lateral) consolidation. Bo et al. (2003) recommended that the required discharge capacity should be in the order of 10^{-6} m/s for 100 mm width drain.

From laboratory tests, the influence factor of time, F_t, can be conservatively estimated to be 1.25. The reduction of the discharge capacity due to bending, folding and twisting has been suggested to be about 48%, giving an influence factor of deformation, F_c of about 2. From filtration tests, the value of F_{fc} is suggested to vary between 2.8 and 4.2 with an average of about 3.5. After considering all the worst conditions that may occur in the field, the discharge capacity, q_w of the PVD could be as high as 500-800 m^3/year, but reduced to 100-300 m^3/year where the hydraulic gradient is unity under elevated lateral pressure (Rixner et al., 1986). The discharge capacity of various types of drains is shown in Fig. 7, where the discharge capacity is influenced by the lateral confining pressure. According to laboratory test data, the discharge capacity can be conservatively assumed to be 100 m^3/year.

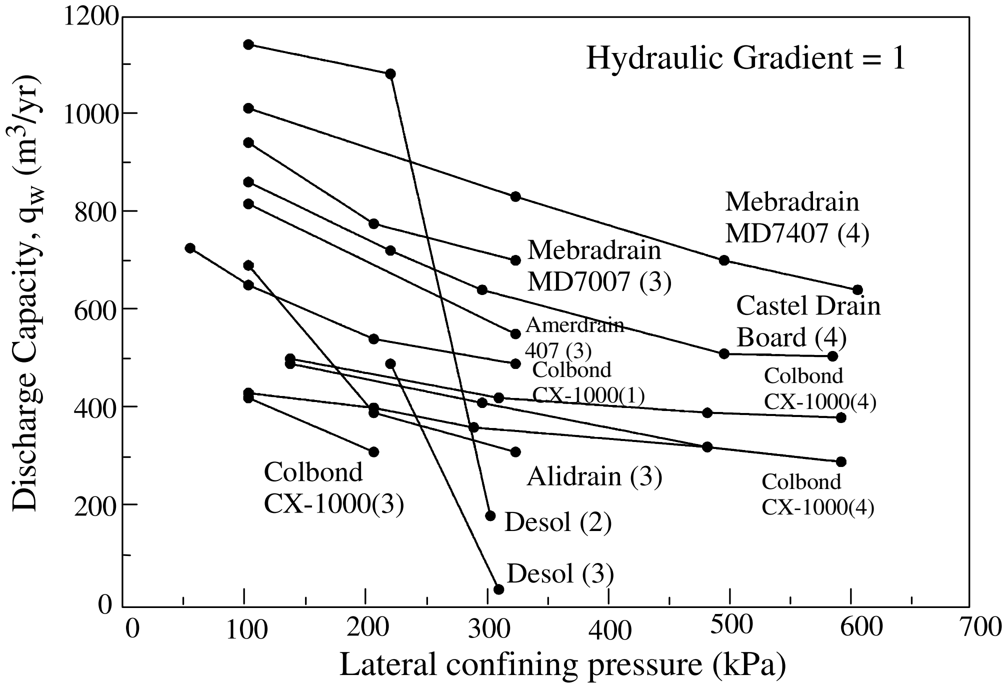


Fig. 7 Typical values of discharge capacity of PVDs (after Rixner et al., 1986)

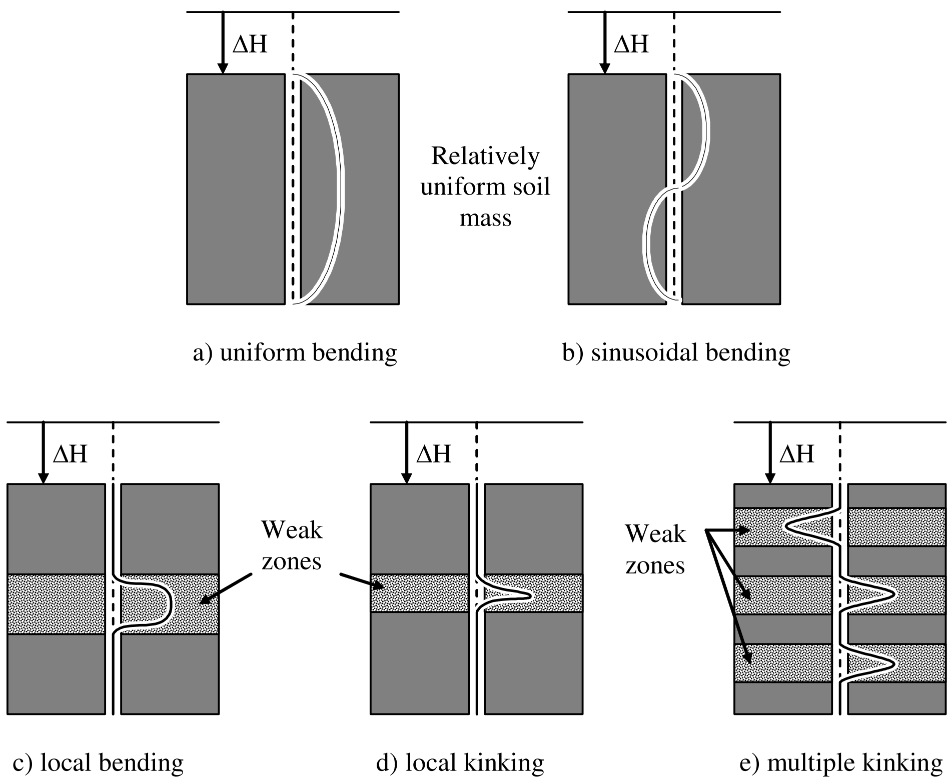


Fig. 8 Possible deformation modes of PVD as a result of ground settlement (adapted from Bergado et al., 1996)

Holtz et al. (1991) suggested that as long as the working discharge capacity of PVD exceeds say 150 m^3/year after installation, the effect on consolidation due to well resistance may not be significant. During recent tests conducted at UoW, it was verified that for PVDs with high discharge capacities, moderately folding and kinking would not affect excess pore water pressure significantly. Indraratna and Redana (2000) described that long term well resistance becomes significant for PVD with q_w less than 40-60 m^3/year. However, discharge capacity q_w can fall below this desired minimum value due to the reasons mentioned earlier. For certain types of PVD, affected by significant vertical compression and high lateral pressure, q_w values may be reduced to 25-100 m^3/year (Holtz et al., 1991). Clearly, the 'clogged' drains are associated with q_w values approaching zero. Hansbo (1979, 1981) presented a closed form solution, which includes the effect of well resistance on drain performance. This factor is a function of the discharge capacity.

Chai et al. (2004) found that the discharge capacity of PVD is a function of the initial discharge capacity (q_o) and hydraulic radius (R) as follows;

$$q = 0.8Rq_0 \tag{11}$$

where, $R = A/L'$, **A** = the cross-sectional area of a drainage channel and L' = the perimeter of the channel.

Smear zone

When vertical drains are pushed into soft ground using a steel mandrel, the surrounding soil is disturbed. These effects due to installation retard the radial consolidation. In this zone, permeability in the smear zone is reduced. The extent of smearing depends on the mandrel size, method of installation and soil type (Eriksson et al., 2000). Jamiolkowski et al. (1981) proposed that the diameter of the smear zone (d_s) and the cross sectional area of mandrel can be related as follows:

$$d_s = \frac{(5 \text{ to } 6)d_m}{2} \tag{12}$$

where, d_m is the diameter of the circle equivalent to area of the mandrel. Hansbo (1981) proposed another relationship as follows:

$$d_s = 2d_m \tag{13}$$

Based on the large-scale laboratory testing (Indraratna and Redana, 1997), the relationship between the diameter of the smear zone (d_s) and the equivalent diameter of the drain can be given by:

$$d_s = (3 \text{ to } 4)d_w \tag{14}$$

Clays with high sensitivity generally exhibit the greatest smear effects. The shape of the smear zone is approximately elliptical around rectangular PVD (Indraratna and Redana, 1998, Welker, 2006). A number of researchers have noted that the disturbance in the smear zone increases towards the drain (Bergado et al., 1991; Hawlader et al., 2002; Sharma and Xiao, 2000; Hird and Moseley, 2000; Indraratna and Redana, 1998; Madhav et al., 1993). Laboratory studies show that the horizontal soil permeability decreases in parabolic shape towards the drain as shown in Fig. 9. The permeability close to the drain is often assumed to be the same as the vertical permeability. The vertical permeability remains relatively unchanged. The ratio of horizontal to vertical permeability (k_h/k_v) approaches unity at the drain soil interface (Indraratna and Redana, 1998).

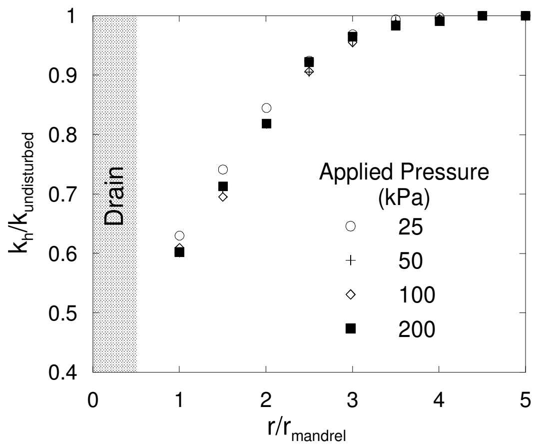


Fig. 9 Variation of horizontal permeability around PVD band drain (Walker, 2006)

The smear zone extent can be quantified either by permeability variation or water content variation along the radial distance (Indraratna and Redana, 1997; Sathananthan and Indraratna, 2006b). Fig. 10 shows the variation of the ratio of the horizontal to vertical permeabilities (k_h/k_v) at different consolidation pressures along the radial distance, obtained from large-scale laboratory consolidation. The variation of the water content with radial distance is shown in Fig. 11. As expected, the water content decreases towards the drain, and also the water content is greater towards the bottom of cell at all radial locations.

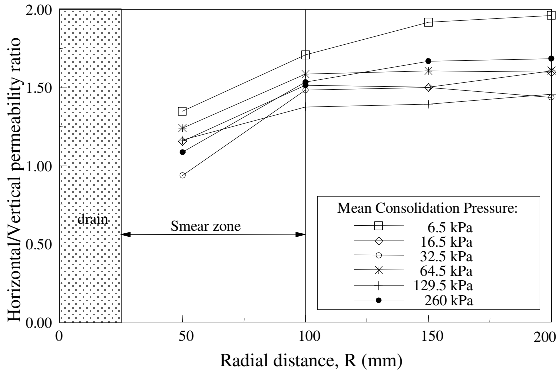


Fig. 10 Ratio of k_h/k_v along the radial distance from the central drain (after Indraratna and Redana 1995)

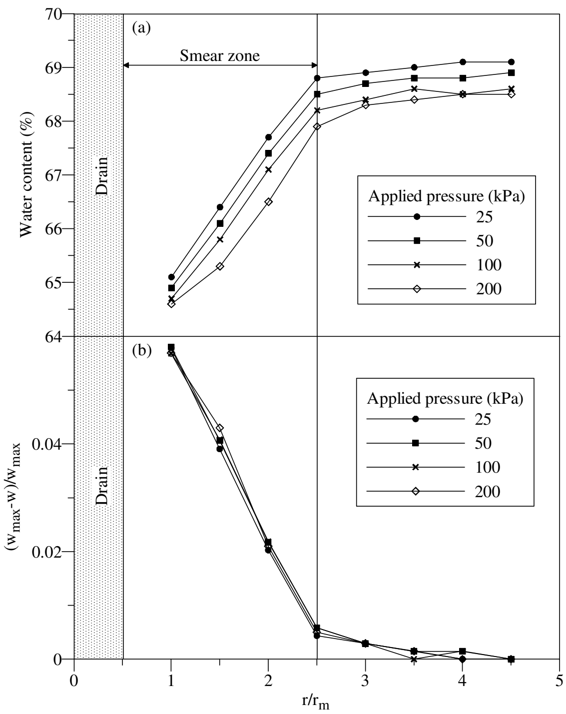


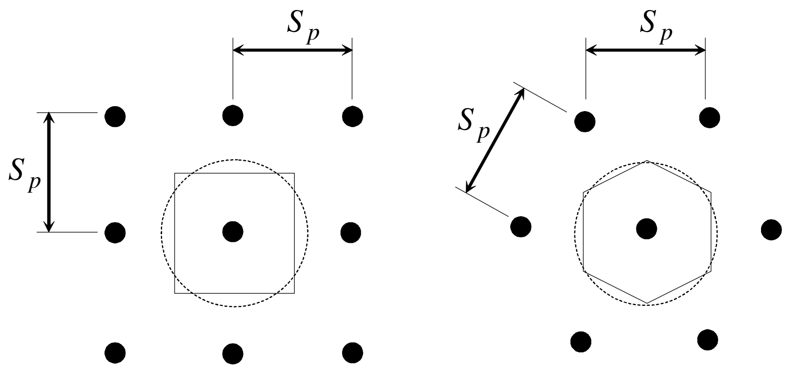
Fig. 11 (a) water content, and (b) normalized water content reduction with radial distance at a depth of 0.5 m (after Sathananthan and Indraratna, 2006b)

Influence zone of drains

Vertical drains are installed in either square or triangular patterns (Fig. 12). The area covered by pore water flowing to a single drain is known as the influence zone. The corresponding diameter of equivalent circular area has to be calculated to convert the square or hexagonal influence zones for the analytical solutions. The corresponding influence radius, r_e for triangular and square spacing arrangements depends on the drain spacing, S_p by:

$$r_e = 0.564 S_p \text{ (Square Pattern)} \tag{15}$$

$$r_e = 0.525 S_p \text{ (Triangular Pattern)} \tag{16}$$



$$r_e = \frac{S_p}{\sqrt{\pi}} = 0.564 S_p \qquad r_e = S_p \sqrt{\frac{\sqrt{3}}{2\pi}} = 0.525 S_p$$

Square pattern Triangular pattern

Fig. 12 Vertical drain installation patterns (Walker, 2006)

A square pattern of drains may be easier to arrange and control during installation in the field, however, a triangular pattern is usually preferred since it provides a more uniform consolidation between drains than the square pattern.

Effect of vacuum removal and reapplication

A large-scale consolidometer was utilized to examine the effect of vacuum preloading in conjunction with surcharge loading (Fig. 13). The results show that a significant increase in settlement rate occurs with the application of vacuum pressure. Also, the settlement associated with combined vacuum and surcharge load indicates the various influences of vacuum removal and re-application during the loading history. The soil properties are given in Table 1.

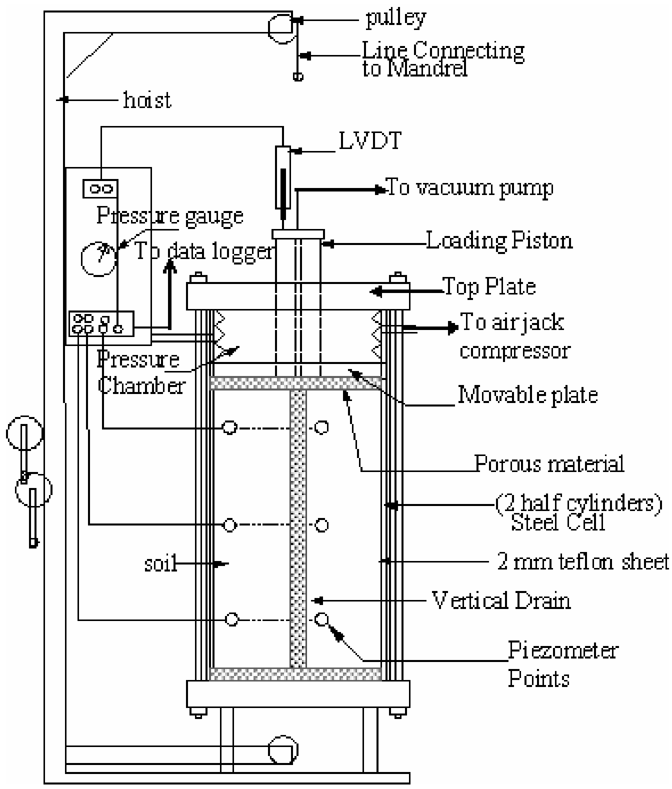


Fig. 13 Schematic of large-scale consolidation apparatus (Indraratna and Redana, 1997)

Table 1 Soil properties of the reconstituted Moruya clay sample (Bamunawita, 2004)

Clay Content (%)	40-50
Silt Content (%)	45-60
Water content, w (%)	40
Liquid Limit, w_L (%)	70
Plastic Limit, w_P (%)	30
Plasticity Index, PI (%)	40
Unit Weight, (t/m^3)	1.81
Specific Gravity, G_s	2.63

The large sacle consolidation test was conducted by applying 100 kPa vacuum pressure to the PVD and the soil surface via the central hole of the rigid piston. Surcharge pressure was applied in two stages 50 kPa and 100 kPa. During a total duration of 28 days, the vacuum pressure was released in two stages for short periods to investigate the effects of vacuum unloading and reloading (Fig. 14a). The measured surface settlements are shown in Fig. 14b. Transducer readings indicate that the maximum measured vacuum pressure is approximately 80 kPa at this depth. The subsequent release of vacuum pressure readily decreases the suction, and the reapplication of vacuum pressure increases the suction rapidly towards 80 kPa again. This also indicates that the suction head decreases with the drain depth, as the maximum suction of 100 kPa could not be maintained along the entire drain length. The settlement associated with combined vacuum and surcharge load is shown in Fig. 14b, and it clearly reflects the effect of vacuum removal and re-application by the corresponding change of gradient of the settlement plot.

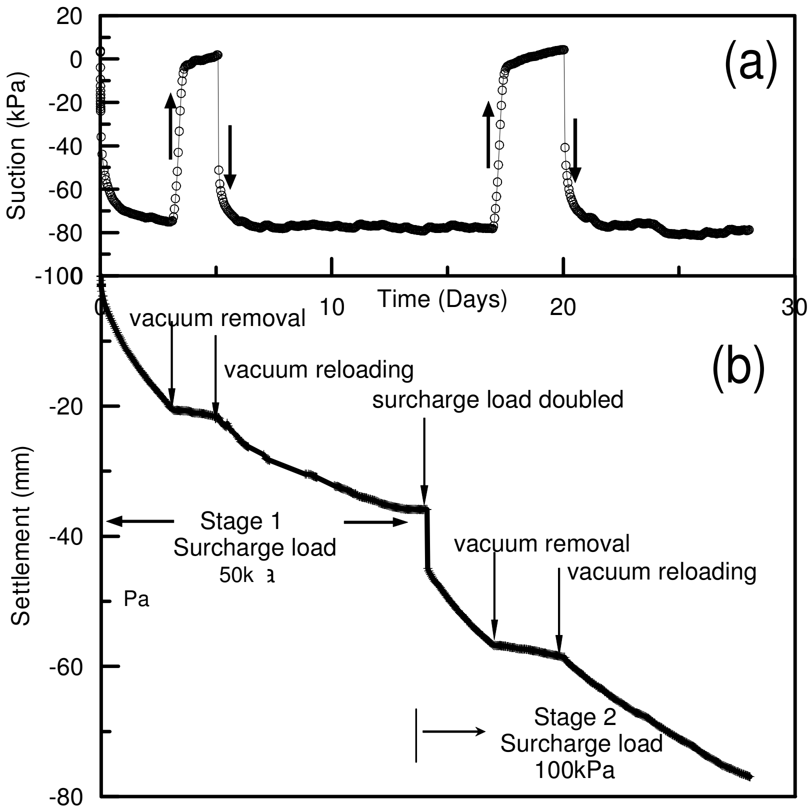


Fig. 14 a) Suction in the drain (240mm from bottom) and b) surface settlement surface settlement associated with simulated vacuum loading and removal (Bamunawita, 2006)

DRAIN UNSATURATION

The non-saturation of the soil adjacent to the drain can occur due to mandrel withdrawal (air gap) and dry conditions of PVDs at the time of installation. The apparent retardation of pore pressure dissipation and consolidation can be found at the initial stage of loading (Indraratna et al. 2004). Figure 15 shows using a numerical simulation, how the top of the drain takes a longer time to be saturated compared to the bottom of the drain. This example assumes that the PVD was 50% saturated at the start. Even for a short drain such as 1m, the time lag for complete drain saturation can be significant.

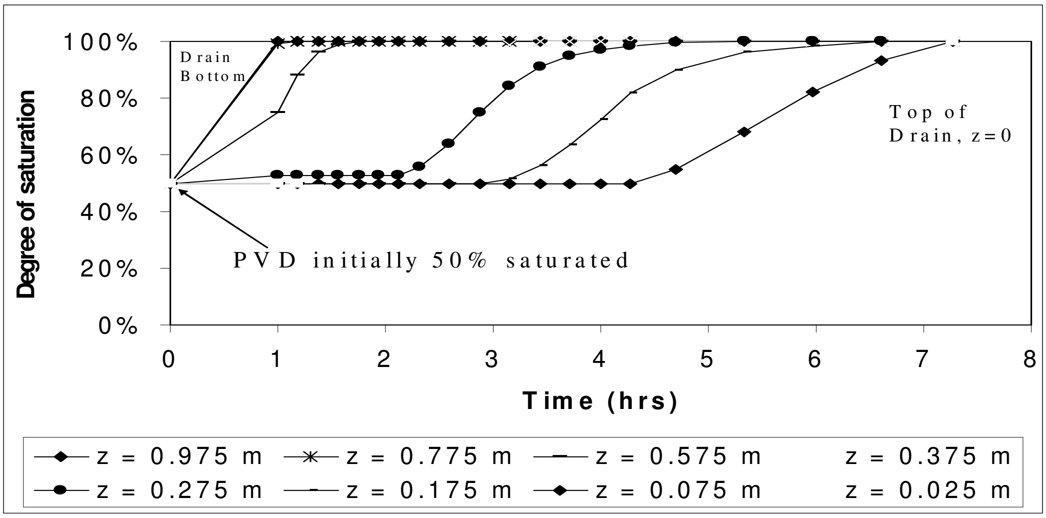


Fig. 15 Degree of drain saturation with time (after Indraratna et al. 2004)

EQUIVALENT PLANE STRAIN ANALYSIS

Indraratna et al. (2005a) showed that based on the appropriate conversion procedure of matching the degree of consolidation at a given time step, the plane stain multi-drain analysis can be employed to predict soft soil behaviour improved by vertical drain and vacuum preloading. To model the radial consolidation, two different approaches are commonly employed; one based on conventional Darcy Law and other based on non-Darcy law (Hansbo 1997). In the first approach, it is assumed that the flow velocity (v) is linearly proportional to the hydraulic gradient (i), i.e. $v = ki$. An exponential flow relationship ($v = \kappa i^n$) is assumed for the second approch (Fig. 16).

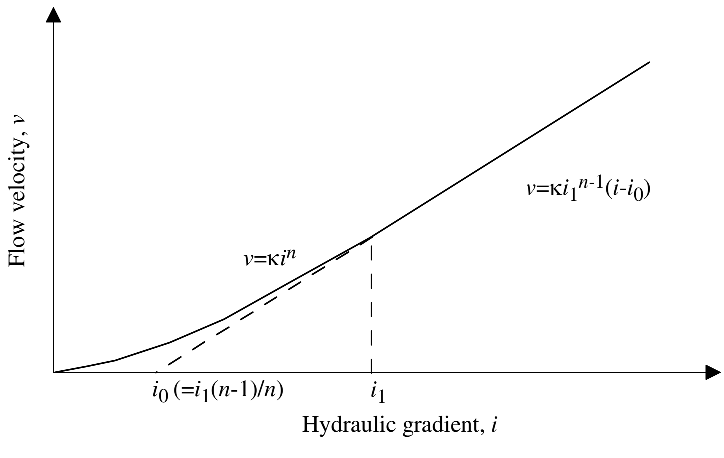


Fig. 16 Exponential correlation (modified after Hansbo, 1997)

Linear Darcian flow:

Using the geometric transformation shown in Fig. 17, the corresponding ratio of the smear zone permeability to the undisturbed zone permeability is obtained by (Indraratna et al., 2005c):

$$\frac{k_{s,ps}}{k_{h,ps}} = \beta / \left(\frac{k_{h,ps}}{k_{h,ax}} \left[\ln\left(\frac{n}{s}\right) + \frac{k_{h,ax}}{k_{s,ax}} \ln(s) - \frac{3}{4} \right] - \alpha \right) \tag{17}$$

where, $\alpha = 0.67 \times (n-s)^3 / n^2(n-1)$ and $\beta = \frac{2(s-1)}{n^2(n-1)}\left[n(n-s-1) + \frac{1}{3}(s^2+s+1) \right]$

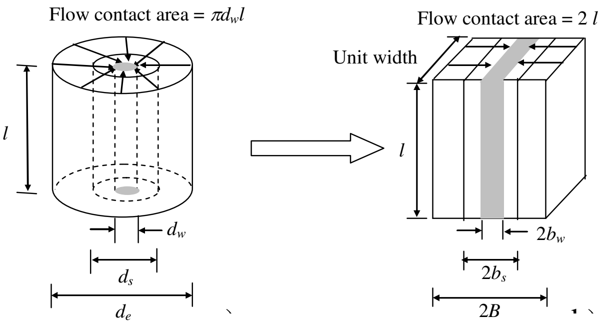


Fig. 17 Unit cell analysis: (a) axisymmetric condition, (b) equivalent plane strain condition (after Indraratna et al., 2005a)

Ignoring both smear and well resistance effects, the simplified ratio of equivalent plane strain to axisymmetric permeability in the undisturbed zone can be attained, hence,

$$k_{h,ps} / k_{h,ax} = \frac{2}{3} \frac{(n-1)^2}{n^2} / \left[\ln(n) - 0.75 \right] \tag{18}$$

The equivalent vacuum pressure can now be expressed by:

$$p_{0,ps} = p_{0,ax} \tag{19}$$

where, k_h = horizontal hydraulic conductivity of soil in the undisturbed zone, k_s = horizontal hydraulic conductivity of soil in the smear zone, T_h = time factor, c_h = coefficient of consolidation for horizontal drainage, n = ratio d_e/d_w (d_e is the diameter of equivalent soil cylinder = $2r_e$ and d_w is the diameter of drain = $2r_w$), s = ratio d_s/d_w (d_s is the diameter of smear zone = $2r_s$.

***Non-Darcian Flow*:**

Sathananthan and Indraratna (2006a) determined the solution for equivalent plane strain under non-Darcian flow. The converted permeability relationship is given by:

$$\kappa_{hp} = 2\kappa_h \left(\frac{n-1}{2n^2} \frac{\beta_p}{\beta} \right)^n \tag{20}$$

where, $\kappa = \left(n^{-1} i^{1-n}\right) k$ (20a)

$$\beta = \frac{1}{3n-1} - \frac{n-1}{n(3n-1)(5n-1)} - \frac{(n-1)^2}{2n^2(5n-1)(7n-1)} + \frac{1}{2n}\left[\left(\frac{\kappa_h}{\kappa_s} - 1\right)\left(\frac{D}{d_s}\right)^{-(1-(1/n))} - \frac{\kappa_h}{\kappa_s}\left(\frac{D}{d_w}\right)^{-(1-(1/n))} \right]$$

(20b)

$$\beta_p = \left(\frac{\kappa_{hp}}{\kappa_{sp}}\right)^{\frac{1}{n}} \left[f_p\left(n, \frac{b_w}{B}\right) - f_p\left(n, \frac{b_s}{B}\right)\right] + f_p\left(n, \frac{b_s}{B}\right) \tag{20c}$$

$$f_p(n, y) = \left[\frac{1}{2!} - \frac{1}{3!\,n} - \frac{(n\text{-}1)}{4!\,n^2} - \frac{(n\text{-}1)(2n-1)}{5!\,n^3} - \frac{(n\text{-}1)(2n-1)(3n-1)}{6!\,n^4} - \ldots\ldots\right]$$

$$- y - \frac{(n\text{-}1)}{2!\,n} y^2 + \frac{(2n\text{-}1)}{3!\,n^2} y^3 + \frac{(n\text{-}1)(3n\text{-}1)}{4!\,n^3} y^4 \tag{20d}$$

$$+ \frac{(n\text{-}1)(2n\text{-}1)(4n\text{-}1)}{5!\,n^4} y^5 + \frac{(n\text{-}1)(2n\text{-}1)(3n\text{-}1)(5n\text{-}1)}{6!\,n^5} y^6 + \ldots\ldots\ldots\ldots\ldots$$

The equivalent plane strain permeability in the undisturbed zone is now obtained as:

$$\frac{\kappa_{hp}}{\kappa_h} = \frac{\lambda_{hp}}{\lambda} = 2\left(\frac{f_p\left(n, \frac{b_w}{B}\right)}{2f\left(n, \frac{r_w}{R}\right)}\right)^n \tag{21}$$

Comparison between the Results of Darcian and Non-Darcian Plane Strain Theories

Sathananthan and Indraratna (2006a) have used the following parameters to study the influence of the exponent n (Eq. 20) on the consolidation process. Consider a plane strain consolidation analysis with the following parameters: B=0.7 m, b_s=0.105 m, b_w=0.035 m, $k_{hp} = \kappa_{hp} = 0.008$ m/yr, $k_{sp} = \kappa_{sp} = 0.002$ m/yr, $c_{hp} = \lambda_{hp} = 0.4$ m^2/yr, and two points (x=0.07 m, within smear zone and x=0.35 m, outside the smear zone) are considered for the comparison of excess pore pressure and hydraulic gradient. In order to demonstrate that the plane strain consolidation equation developed for non-Darcian flow is also valid for Darcian flow, i.e. when $n \to 1$, an example is given below.

Figure 18 shows that the average degree of consolidation increases from about 52% to 68% when n varies from 1.00 to 1.50. In other words, when n increases from 1 to 1.5, the degree of consolidation is increased by approximately 30%. Practically speaking this is a significant deviation from conventional consolidation theory, hence emphasising the influence of the exponent n.

SALIENT ASPECTS OF NUMERICAL MODELLING

Currently, sophisticated finite element softwares have been used in the design to predict the field performance such as pore pressures, settlements, lateral displacements, and stresses of the field site with vertical drains. Commercial software packages such as PLAXIS, ABAQUS, and SAGE-CRISP are capable of performing fully coupled consolidation analysis. In this section, the recent finite element models applied to vertical drains are described below.

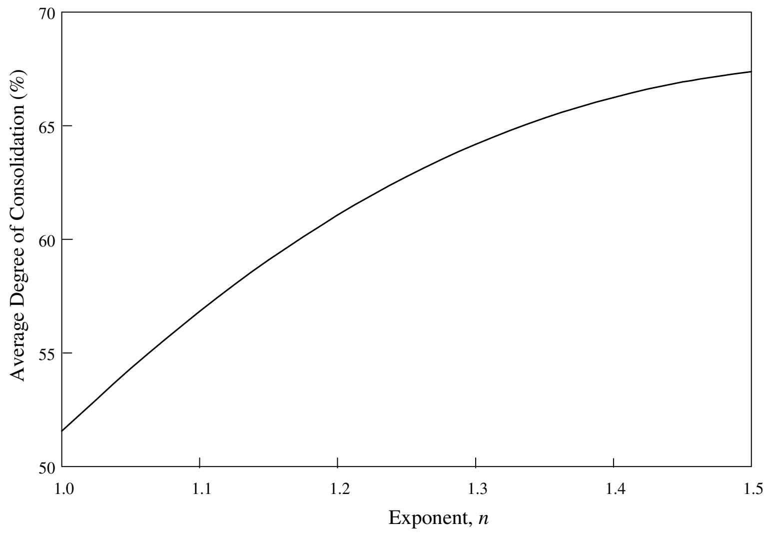


Fig. 18 Variation of average degree of consolidation with exponent, *n* after 0.5 years

(Sathananthan and Indraratna 2006a)

PERFORMANCE OF TEST EMBANKMENT STABILISED WITH VERTICAL DRAINS ON SOFT CLAY

As part of the Second Bangkok International Airport (30 km east of Bangkok, Thailand) a series of test embankments was constructed to assess the behaviour of the thick compressible subsoil. The site was occupied by ponds for fish farming and for agricultural purposes in the past. In this area, soft clays, mainly of marine or deltaic origin, often present substantial construction problems, which require ground improvement techniques to eliminate excessive settlement and lateral movement. Two embankments were analysed, one with PVDs alone and the other with PVDs plus vacuum preloading.

The subsoil is relatively uniform and consists of a top weathered clay crust (2 m depth) underlain by soft to medium layers that extend to 10 m depth. Underneath the medium clay layer, a stiff clay layer is established at a depth of 10-21m. The ground-water level was at the surface. The parameters of subsoil layers based on laboratory testing are given in Table 2.

Table 2 Selected soil parameters in FEM analysis

Depth (m)	λ	κ	ν	e_0	γ kN/m^3	k_v 10^{-9} m/s	k_h 10^{-9} m/s	k_s 10^{-9} m/s	k_{hp} 10^{-9} m/s	k_{sp} 10^{-9} m/s
0.0-2.0	0.3	0.03	0.30	1.8	16	15.1	30.1	89.8	6.8	3.45
2.0-8.5	0.7	0.08	0.30	2.8	15	6.4	12.7	38.0	2.9	1.46
8.5-10.5	0.5	0.05	0.25	2.4	15	3.0	6.0	18.0	1.4	0.69
10.5-13.0	0.3	0.03	0.25	1.8	16	1.3	2.6	7.6	0.6	0.30
13.0-15.0	1.2	0.10	0.25	1.2	18	0.3	0.6	1.8	0.1	0.07

Embankment with PVDs alone

The test embankment (TS1) was constructed in stages, with PVD spacings of 1.5 m in a square pattern, installed to a depth of 12 m. The test embankment is 4.2 m high, 7.4m × 7.4m at the top, and 40 m x 40 m at the bottom with 3H:1V side slope. A section of test embankment TS1, which is stabilized with Flodrain (FD4-EX) is shown in Fig. 19 together with the subsoil variation and the vertical drain pattern.

The construction of embankment TS1 was carried out in 4 stages. During the first 15 days (Stage 1), a 1.0 m thick sand blanket with a surcharge of 18 kPa was laid on the ground. After a rest period of 30 days, the PVDs were installed followed by 1.0m clayey sand in 10 days (Stage 2) with a surcharge of 45 kPa. The embankment thickness was increased to 3.0m (54 kPa) in 10 days (Stage 3) after a rest period of 90 days. Followed by another rest period of 45 days, the height of the embankment was raised to 4.2m (75 kPa) in 25 days (Stage 4). To maintain stability of the embankment, an additional berm 5m wide and 1.5m high was included in Stage 3. The berm width was increased to 7m in the final Stage.

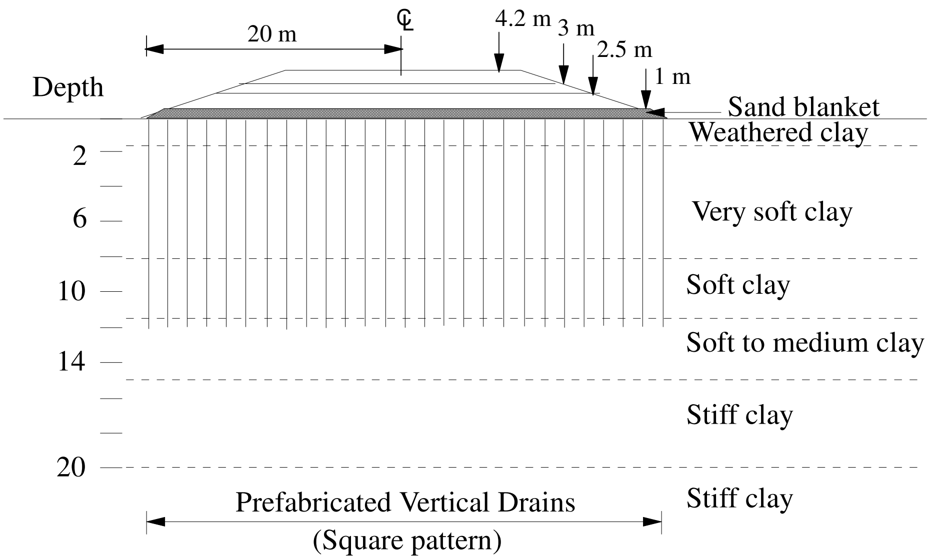


Fig. 19 Cross section at embankment with sub-soil profile, Second Bangkok International Airport, Thailand (Redana, 1999)

Embankment with PVDs and vacuum preloading

Embankment TV2 was constructed with PVDs and vacuum application. PVDs with 12m long were installed below a hypernet drainage system. The drainage blanket (working platform) was constructed with sand 0.8 m with an air and water tight Linear Low Density Polyethylene (LLDPE) geomembrane liner placed on top of the drainage system. This liner was sealed by placing its edges at the bottom of the perimeter trench and covered with a 0.3m layer of bentonite and then submerged with water. The array of instrumentation included piezometers, surface settlement plates, multipoint extensometers, inclinometers, observation wells and benchmarks (Fig. 20).

During the application of vacuum pressure, it was found that the suction head transmitted to the soil could not be maintained at the same level throughout the vacuum pressure application period as shown in Fig. 21 (Sangmala, 1997). This fluctuation has not been uncommon in various soft clays, and has often been associated with air leaks through the surface membrane or the loss of suction head beneath the certain depth for long PVD.

Intersection of natural macro-pores with drains at various depths can also lead to suction head drops.

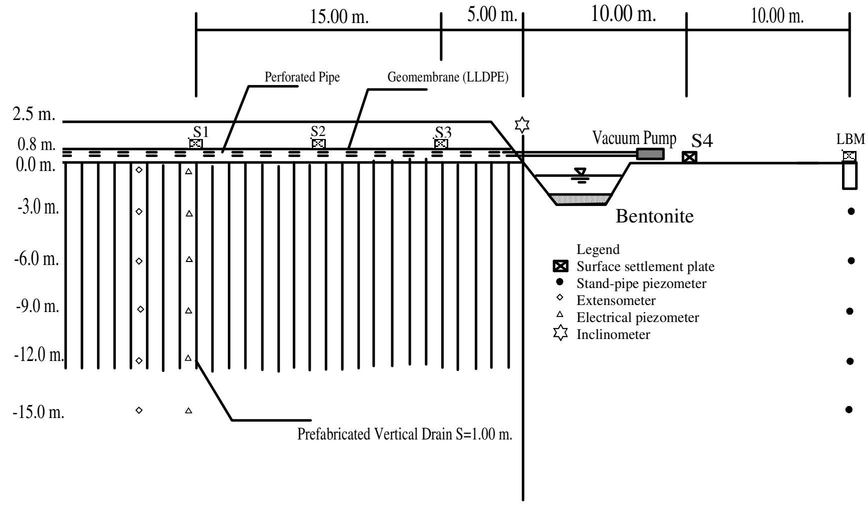


Fig. 20 Vertical cross section at embankment TV2 with instrumentation locations (adapted from Indraratna et al. 2005b)

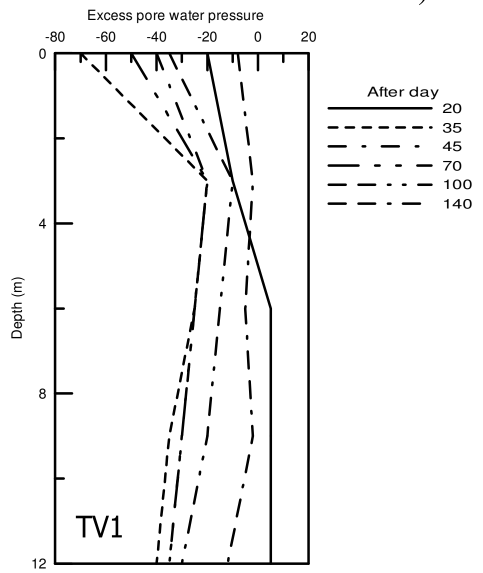


Fig. 21 Observed variation of pore pressure with time and depth in the field for two embankments TV1

Multi-drain Analysis by Equivalent Plane Strain Model Incorporating Finite Element Analysis

The consolidation behaviour of soft clay beneath the embankments combined with vacuum and surcharge preloading was analysed using the finite element software ABAQUS. The equivalent plane strain model (Eqs. 17-19) as well as the modified Cam-clay theory (Roscoe and Burland, 1968) were incorporated in the analysis. The parameters of subsoil layers based on laboratory testing are given in Table 2. The equivalent vertical band drain radius r_w was determined using the method proposed by Eq. (1), which gives r_w =0.03m. According to Indraratna and Redana (1997), the ratios of k_h/k_s and d_s/d_w determined in the laboratory are approximately 1.5-2.0 and 3-4, respectively, but in practice these ratios can vary from 1.5 to 6 depending on the type of drain, mandrel size and shape and installation procedures used (Indraratna and Redana, 2000; Saye, 2003). The values of k_h/k_s and d_s/d_w (Table 2) for this case study were assumed to be 2 and 6, respectively (Indraratna et al., 2004). For the plane strain FEM simulation, the equivalent permeability inside and outside the smear zone was determined using Eqs. 15 and 16. The discharge capacity is assumed high enough and can be neglected (Indraratna and Redana, 2000).

The finite element mesh contained 8-node bi-quadratic displacement and bilinear pore pressure elements (Fig. 21). Only the right hand side of the embankment was modeled due to symmetry, as shown in Fig. 22. For the PVD zone and smear zone, a finer mesh was implemented so that each unit cell represented a single drain with the smear zone established on either side. The different drain lengths for the two embankments were modelled by fixing the pore pressure boundary along the appropriate depths. The incremental surcharge loading was simulated at the upper boundary.

The backfill soil material used in the embankment was compacted clayey sand. As the Mohr-Coulomb (M-C) failure criteria represent the failure of soil having an apparent cohesion, the elastic-perfectly plastic, M-C model was considered appropriate for representing the embankment fill (Indraratna and Sathananthan, 2004). The uppermost 2 m of the soil profile consists of heavily overconsolidated (HOC) weathered clay (crust). Since the Cam Clay model is unsuitable for the simulation of HOC soil, M-C model has been used to represent the upper crust also. For the normally consolidated and lightly overconsolidated clay layers, the Cam Clay model is appropriate.

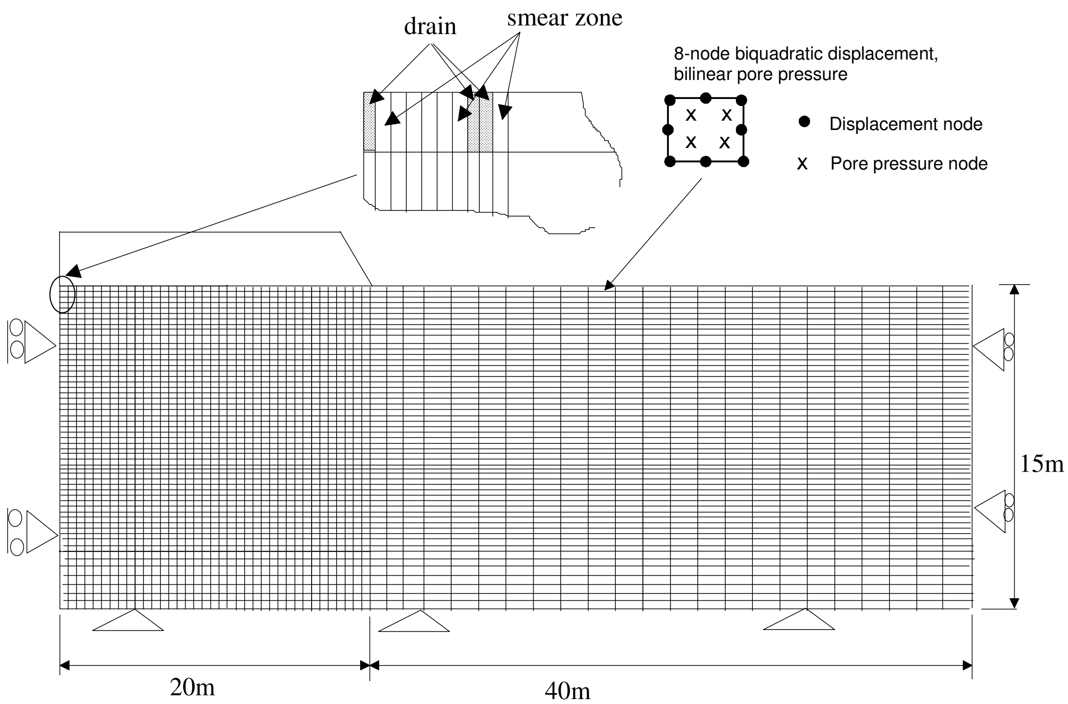


Fig. 22 Finite element mesh for multi-drain analysis (Indraratna et al. 2005b)

The predictions of degree of consolidation based on settlement at the embankment centreline are shown in Fig. 23. The analysis employing the proposed conversion procedure including smear effects could predict the field data accurately. It can be seen that the time required to achieve the desired settlement can decrease from 400 days to 120 days with the application of vacuum pressure by avoiding the need for staged construction (Indraratna et al., 1992). Figure 24 presents the time dependent excess pore water pressure. The vacuum loading generated negative excess pore pressures, while the embankment with surcharge fill created positive excess pore pressure. The excess pore pressure predictions agree well with the field results. It can be seen that the maximum magnitude of the measured negative excess pore pressure is about 35 kPa due to the vacuum loss along the drain length and possible air leak from the membrane. The reduction in the negative pore pressure magnitude at various times is due to the removal of the vacuum application.

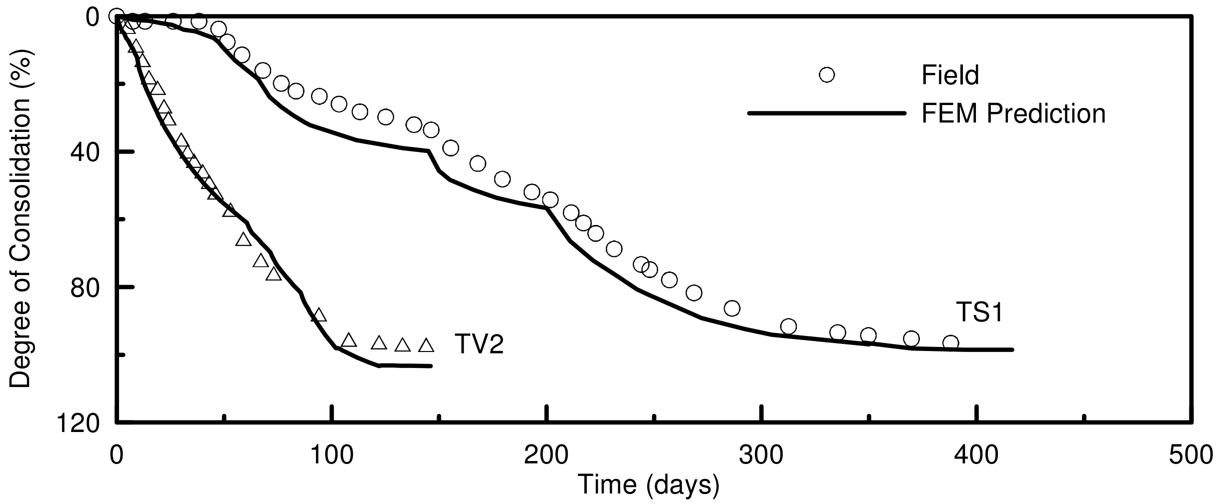


Fig. 23 Degree of Consolidation at the centreline for embankments (after Indraratna and Redana, 2000 and Indraratna et al., 2005c)

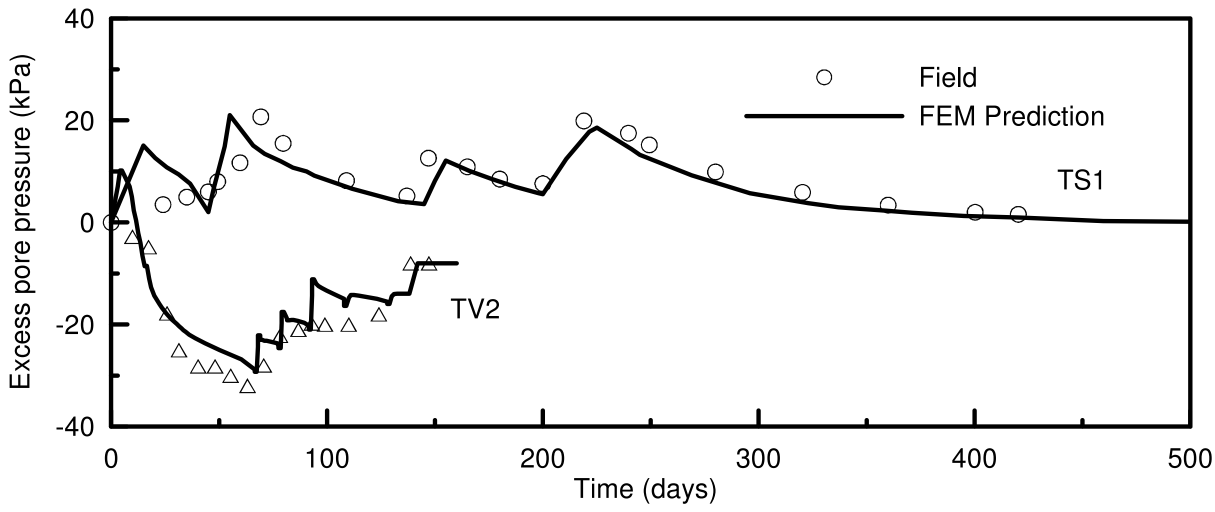


Fig. 24 Excess pore pressure variation (after Indraratna and Redana, 2000 and Indraratna et al., 2005d)

The comparisons between predicted and measured lateral movement at the end of construction for embankments TS1 and TV2 are presented in Fig 25. As discussed by Indraratna et al. (2005d), the stiff behaviour of the crust cannot be modelled using conventional Cam-Clay properties, and it needs to be modelled as a highly over-consolidated (compacted) layer. In this analysis, the Mohr Coulomb model was used to represent the Over-consolidated crust. From the field measurement, the observed lateral displacements of the crust (0-2m) are similar to the numerical predictions. It shows that a heavily overconsolidated crust makes a favourable contribution in controlling the lateral yield below the embankment base. The plane strain FEM model enables good prediction of the lateral yield beneath both embankments. Comparison between the cases of with and without vacuum pressure clearly indicates that vacuum preloading reduces an outward lateral movement. It is clearly shown that the constitutive model of the compacted crust has to be selected properly in the FEM analysis to obtain an accurate prediction.

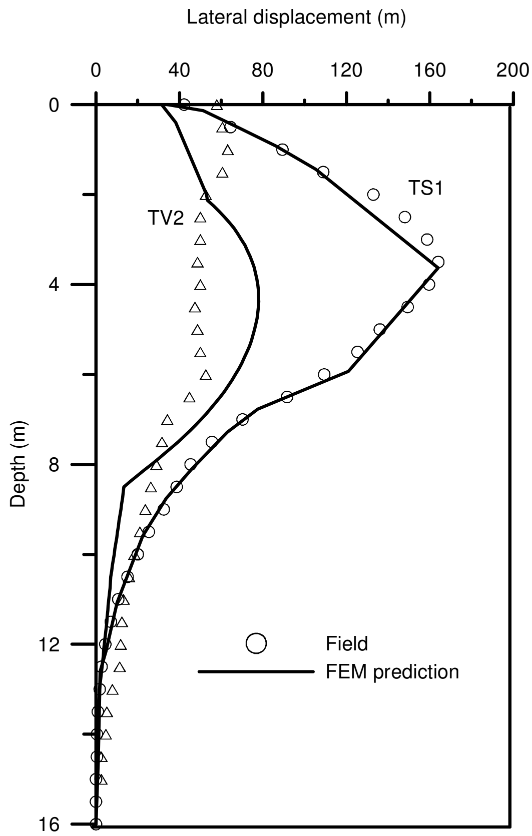


Fig. 25 Calculated and measured lateral displacements at the end of construction at the embankment toe (after Indraratna and Redana, 2000 and Indraratna et al., 2005c)

SKA-EDEBY EMBANKMENT, STOCKHOLM, SWEDEN

Indraratna and Sathananthan (2006) demonstrate the practical application of non-Darcian plane strain solution through a case history at Ska-Edeby, 25 km west of Stockholm, Sweden. The site details including the construction history and soil parameters are given elsewhere by Hansbo (1997). Area II with 180 mm diameter sand drains in an equilateral triangular pattern at 1.5 m spacing (i.e. D=1.58 m) with an equivalent loading of 32 kPa is analysed. Using the trial and error method, the following parameters are selected for axisymmetric analysis (Hansbo, 1997): d_s=0.36m, $k_h/k_s = \kappa_h/\kappa_s = 4$, c_h=1.10 m^2/year λ =0.54 m^2/year and n=1.5.

In Fig. 26, the estimated degree of consolidation based on the Darcian axisymmetric, non-Darcian axisymmetric (Hansbo, 1997) and non-Darcian plane strain solutions (Sathananthan and Indraratna, 2006) are plotted with the available field data at embankment centerline. The predicted values based on non-Darcian flow seem to agree better with the field data in relation to the Darcian (conventional) analysis. However, in the opinion of the authors, this difference is usually small, and for all practical purposes the conventional Darcy conditions are still sufficient.

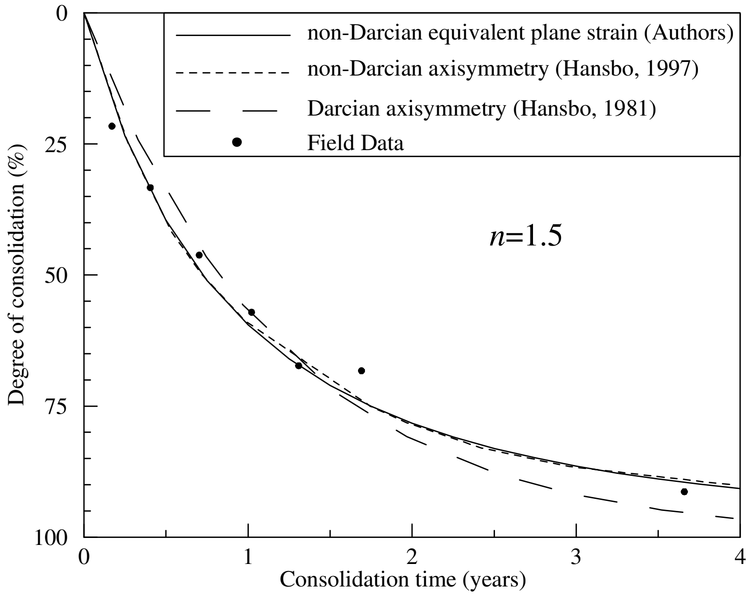
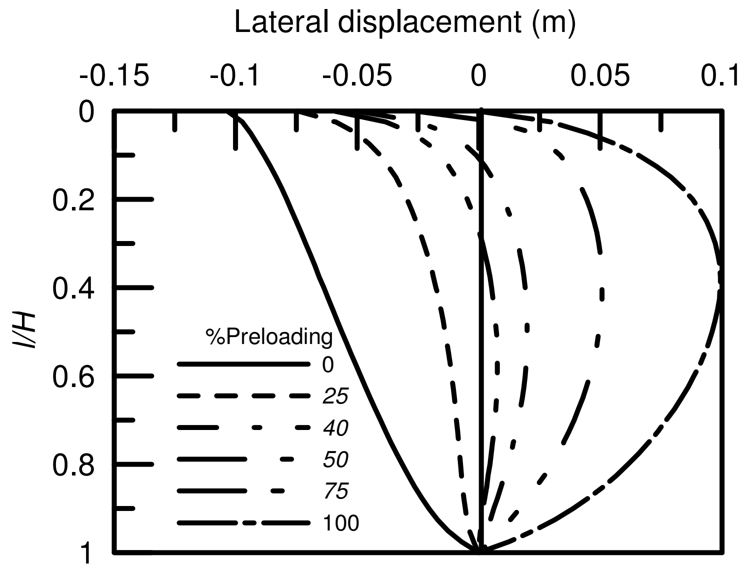


Fig. 26 Degree of consolidation at the embankment centreline with time for Area II, Ska-Edeby field study (after Hansbo, 1997; Sathananthan & Indraratna, 2005)

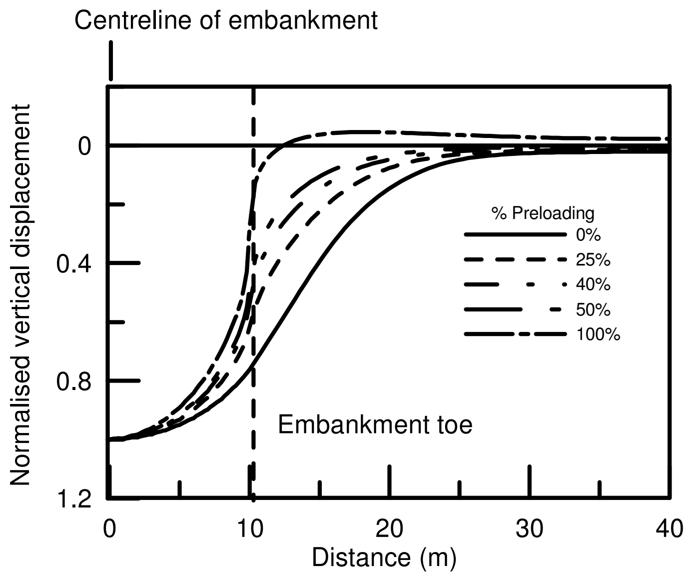
EFFECTS ON LATERAL YIELD OF SOFT CLAYS DUE TO VACUUM CONSOLIDATION

In order to investigate the effects of vacuum pressure combined with surcharge load on the lateral displacements, a simplified plane strain (2-D) finite element analysis was adopted. The outward lateral compressive strain due to surcharge can be reduced by the application of suction (vacuum preloading). However, excessive inward lateral movement may sometimes generate tension cracks in the adjacent areas. The variation of vacuum and preloading pressure to obtain a given required settlement will be considered in the numerical model to optimise the lateral displacement at the embankment toe, while identifying any zones of tension.

The variation of lateral displacement at the embankment toe due to various preloading pressures (total preloading pressure of 150 kPa) is shown in Fig. 27a. The negative lateral displacement represents an inward soil movement towards the centreline of the embankment. As expected, the vacuum application alone can create the maximum inward lateral movement, whereas preloading without any vacuum pressure may contribute to the maximum outward lateral movement. For a uniform soil layer, the combination of 40% surcharge preloading stress with 60% vacuum pressure seems to maintain the lateral displacements close to zero axis (i.e., an optimum balance between vacuum preloading and surcharge preloading for the given soil properties. Fig. 27b presents the variation of surface settlement profiles with increasing % surcharge loading. The effect of vacuum pressure alone may create settlements up to 10m away from the embankment toe. Also, due to the outward lateral displacement, soil heave can be observed beyond the toe.



(a)



(b)

Fig. 27 (a) Lateral displacements and (b) Surface settlement profiles (Indraratna and Rujikiatkamjorn. 2008)

CONCLUSION

Various types of vertical drains have been used to accelerate the rate of primary consolidation. With vacuum application, it is anticipated that with an effective airtight membrane placed over the surface, the applied suction head will propagate along the surface and down the drains, consolidating and strengthening the soil within the PVD stabilised zone. Also, the thickness of the surcharge fill required otherwise may be reduced by several meters, if sufficient vacuum pressure (less than 100 kPa) is applied and sustained, thereby reducing the risk of undrained bearing capacity failure. Once the soil has undergone initial settlement (increased shear strength), the post-construction soil deformation will be considerably less, thereby eliminating any risk of instability of the overlying infrastructure.

For large construction sites, the plane strain analysis is often sufficient to obtain accurate predictions. A finite element code (ABAQUS) was used to analyse the behaviour of soft soil improved by PVDs and compared with field measurements. The equivalent plane strain solution was applied for selected case histories, demonstrating its validity in predicting the real behaviour. Field behaviour as well as model predictions indicate that the efficiency of vertical drains depends on the magnitude and distribution of vacuum pressure as well as the degree of drain saturation during installation. The field application of the non-Darcian plane strain solution is demonstrated on the basis of a well documented pilot test taken from Ska-Edeby, Sweden. The results clearly show that the predicted values based on the non-Darcian flow are similar to the conventional analysis.

The accurate prediction of lateral displacements and pore pressures are often difficult tasks compared to settlements. This may be attributed to the complexity of evaluating the true magnitudes of soil parameters inside and outside the smear zone, correct drain properties as well as the capabilities of the soil constitutive model. Therefore, one needs to use the most appropriate laboratory techniques to obtain parameters, preferably using large-scale testing equipment. The smear zone radius was 2-3 times the radius of the mandrel. The soil permeability in the smear zone is higher than that in the undisturbed zone by a factor of 1.5-2. The drain unsaturation can delay the consolidation during the initial stages. The accurate prediction of lateral displacement at shallow depths depends on the correct assessment of soil properties including the over-consolidated surface crust. This compacted layer is relatively stiff, and therefore it resists 'inward' movement of the soil upon the application of vacuum pressure. The Mohr Coulomb model can be used as an appropriate model for modelling the weathered and compacted crust. The analysis of Bangkok case histories shows that the vacuum application via PVD significantly decreases lateral displacement. As a result, the rest period during embankment construction can be avoided.

There is no doubt that a system of vacuum consolidation via PVDs is a useful and practical approach for accelerating radial consolidation. Such a system eliminates the need for huge amount of good quality surcharge material, via air leak protection in the field. Accurate modelling of vacuum preloading requires both laboratory and field studies to quantify the nature of vacuum pressure distribution within a given formation and drain system.

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REFERENCES:

Atkinson, M.S. and Eldred, P.J.L. (1981). Consolidation of soil using vertical drains. Geotechnique, Vol. 31, No. 1, pp. 33-43.

Bamunawita, C. (2004). Soft clay foundation improvement via prefabricated vertical drains and vacuum preloading. PhD thesis, University of Wollongong, Wollongong, New South Wales, Australia.

Bergado, D.T., Asakami, H., Alfaro, M.C., and Balasubramaniam, A.S. (1991). Smear effects of vertical drains on soft Bangkok clay. Journal of Geotechnical Engineering, ASCE, Vol.117, No. 10, pp. 1509-1530.

Bergado, D.T., Manivanna, R. and Balasubramaniam, A.S. (1996). Proposed criteria for discharge capacity of prefabricated vertical drains. Geotextiles and Geomembranes, 14(9 Sep), pp. 481-505.

Bo, M. W., Chu, J., Low, B. K., and Choa, V. (2003). Soil improvement; prefabricated vertical drain techniques, Thomson Learning, Singapore.

Carroll, R. G. (1983). Geotextile filter criteria. Transportation research record, 916, pp. 46-53.

Chai, J. C., Miura, N., and Nomura, T. (2004). Effect of hydraulic radius on long-term drainage capacity of geosynthetics drains. Geotextiles and Geomembranes, Vol. 22, pp. 3-16.

Christopher, B. R., and Holtz, R. D. (1985). Geotextile Engineering Manual. FHWA-TS-86/203, U.S. Federal Highway Administration.

Chu, J., M.W. Bo and Choa, V. (2004). Practical considerations for using vertical drains in soil improvement projects. Geotextiles and Geomembranes, Wol. 22(1-2 February/April), pp. 101-117.

Eriksson, U., Hansbo, S. and Torstensson B.-A. (1999). Soil improvement at Stockholm-Arlanda Airport. Ground Improvement, Vol. 4, No. 2, pp. 73-80.

Fellenius, B.H. and Castonguay, N.G. (1985). The efficiency of band shaped drains - a full scale laboratory study, National Research Council of Canada and the Industrial Research Assistance Programme, 54.

Hansbo, S. (1979). Consolidation of clay by band-shaped prefabricated drains. Ground Eng., Vol. 12, No. 5, pp. 16-25.

Hansbo, S. (1981). Consolidation of fine-grained soils by prefabricated drains. In Proceedings of 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Balkema, Rotterdam, 3, pp. 677-682.

Hansbo, S. (1997). Aspects of vertical drain design — Darcian or non-Darcian flow. Géotechnique Vol. 47, No. 5, pp. 983–992.

Hird, C.C. and V.J. Moseley (2000). Model study of seepage in smear zones around vertical drains in layered soil. Geotechnique, Vol. 50, No. 1, pp. 89-97.

Holtz, R.D., Jamiolkowski, M., Lancellotta, R. and Pedroni, S. (1991). Prefabricated vertical drains: design and performance, CIRIA ground engineering report: ground improvement. Butterworth-Heinemann Ltd, UK, 131 p.

Indraratna B., and Chu J. (2005). Ground Improvement – Case Histories Book (Volume 3), Elsevier, London 1115p.

Indraratna B., and Redana I.W. (1997). Plane strain modelling of smear effects associated with vertical drains, Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 123. No. 5, pp. 474-478.

Indraratna, B., and Redana, I. W. (1998). Laboratory determination of smear zone due to vertical drain installation. J. Geotech. Eng., ASCE, Vol. 125 No. 1, pp. 96-99.

Indraratna, B., and Redana, I. W. (2000). Numerical modelling of vertical drains with smear and well resistance installed in soft clay. Canadian Geotechnical Journal, Vol. 37, pp. 132-145.

Indraratna, B, and Rujikiatkamjorn, C. (2008). Effects of partially penetrating prefabricated vertical drains and loading patterns on vacuum consolidation, GeoCongress 2008, New Orleans, (9-12 March 2008) (Accepted).

Indraratna, B. and Sathananthan, I. (2004). Numerical Prediction of Soft Clay Consolidation with Geosynthetic Vertical Drains Using Plane Strain Solution. 9th ANZ Conference on Geomechanics, Auckland, New Zealand, Vol. 2, pp. 633-639.

Indraratna, B., Balasubramaniam, A. S. and Balachandran, S. (1992). Performance of test embankment constructed to failure on soft marine clay. Journal of Geotechnical Engineering, ASCE, Vol. 118, No. 1, pp. 12-33.

Indraratna, B., Balasubramaniam, A. S., and Ratnayake, P. (1994). Performance of embankment stabilized with vertical drains on soft clay. J. Geotech. Eng., ASCE, Vol. 120, No. 2, pp. 257-273.

Indraratna, B., Bamunawita, C., and Khabbaz, H. (2004). Numerical modelling of vacuum preloading and field applications. Canadian Geotechnical Journal, Vol. 41, pp. 1098-1110.

Indraratna, B., Rujikiatkamjorn C., and Sathananthan, I., (2005a). Analytical modelling and field assessment of embankment stabilized with vertical drains and vacuum preloading. The Proceedings of the 16th International Conference on Soil Mechanics and Geotechnical Engineering, 12-16 September 2005, Osaka, Japan, Edited by the 16th ICSMGE committee, Millpress, Rotterdam, the Netherlands. pp. 1049-1052.

Indraratna, B., Rujikiatkamjorn C., and Sathananthan, I. (2005b). Analytical and numerical solutions for a single vertical drain including the effects of vacuum preloading. Canadian Geotechnical Journal, Vol. 42, pp. 994-1014.

Indraratna, B., Rujikiatkamjorn C., and Sathananthan, I. (2005c). Radial consolidation of clay using compressibility indices and varying horizontal permeability. Canadian Geotechinical Journal, Vol. 42. pp. 1330-1341.

Indraratna, B., Rujikiatkamjorn C., Balasubramaniam, A. S. and Wijeyakulasuriya, V. (2005d). Predictions and observations of soft clay foundations stabilized with geosynthetic drains and vacuum surcharge. Ground Improvement – Case Histories Book (Volume 3), Edited by Indraratna, B. and Chu, J., Elsevier, London, pp. 199-230.

Indraratna, B., Sathananthan, I., Rujikiatkamjorn C. and Balasubramaniam, A. S. (2005e). Analytical and numerical modelling of soft soil stabilized by PVD incorporating vacuum preloading. International Journal of Geomechanics, Vol. 5 No. 2, pp. 114-124.

Jamiolkowski, M., Lancellotta, R., and Wolski, W. (1983). Precompression and speeding up consolidation. Proc. 8th ECSMFE, pp. 1201-1206.

Kjellman, W. (1948). Accelerating consolidation of fine grain soils by means of cardboard wicks. Proc. 2nd ICSMFE, 2, pp. 302-305.

Koerner, R.M., Ed. (1987). Soft soil stabilization using geosynthetics. London, Elsevier Applied Science.

Long, R.P. and Covo, A. (1994). Equivalent diameter of vertical drains with an oblong cross section. J. Geotech. Eng. Div., ASCE, Vol. 120, No. 9, pp. 1625-1630.

Madhav, M.R., Park, Y.-M. and Miura, N. (1993). Modelling and study of smear zones around band shaped drains. Soils and Foundations, 33(4), 135-147.

Miura, N., and Chai, J. C. (2000). Discharge capacity of prefabricated vertical drains confined in clay. Geosynthetics International, Vol. 7. No. 2, pp. 119-135.

Redana, I. W. (1999). Effectiveness of vertical drains in soft clay with special reference to smear effect. PhD Thesis University of Wollongong.

Rixner, J.J., Kraemer, S.R. and Smith, A.D. (1986). Prefabricated Vertical Drains, Vol. I, II and III: Summary of Research Report-Final Report. Federal Highway Admin., Report No. FHWA-RD-86/169, Washington D.C, 433 p.

Roscoe, K.H., and Burland, J.B. (1968). On the generalized stress strain behaviour of wet clay. Engineering plasticity, Cambridge Univ. Press; Cambridge, U.K., pp. 535-609.

Sathananthan, I. (2005). Modelling of Vertical Drains with Smear Installed in Soft Clay. PhD Thesis, University of Wollongong, 264p.

Sathananthan, I. and Indraratna, B. (2006a). Plane Strain Lateral Consolidation with Non-Darcian Flow. Canadian Geotechnical Journal, Vol. 43, pp. 119-133.

Sathananthan, I. and Indraratna, B. (2006b). Laboratory Evaluation of Smear Zone and Correlation between Permeability and Moisture Content. Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 132. No. 7, pp. 942-945.
Sangmala, S. (1997). Efficiency of drainage systems of vacuum preloading with surcharge on PVD improved soft Bangkok clay, ME Thesis, Asian Institute of Technology, Bangkok, Thailand.
Saye, S.R. (2001). Assessment of soil disturbance by the installation of displacement sand drains and prefabricated vertical drains. Geotechnical Special Publication #119, ASCE, pp. 325-362.
Seah, T.H. (2006). Design and construction of ground improvement works at Suvarnabhumi Airport. Geot. Eng, J. of Southeast Asian Geot. Society, Vol. 37, pp. 171-188.
Walker, R. (2006). Analytical Solutions for Modelling Soft Soil Consolidation by Vertical Drains. PhD thesis, University of Wollongong, Wollongong, New South Wales, Australia.